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# THEORETICAL ELASTIC MISMATCH STRESSES

*by Robert H. Johns*

*Lewis Research Center*

*Cleveland, Ohio*



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# THEORETICAL ELASTIC MISMATCH STRESSES

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## SUMMARY

Start

Stress-magnification factors were obtained for the effect of mismatch of the middle surfaces at junctions in shell structures. The results are directly applicable only when the thickness and the slope on one side of a mismatched joint are the same as those on the other. Results are found for biaxial stress states with any principal stress ratio at the joint, including cases in which the stresses are of opposite sign. Both principal and effective stress-magnification factors are given for long or local mismatch. The results are presented both graphically and in equation form. → 22 to p. 22

As expected, the increase in stress is substantially higher when the membrane stress normal to the mismatch is greater in magnitude than that parallel to the mismatch. The effective stress factors are highest when the membrane stress normal to the mismatch is about twice the membrane stress parallel to the mismatch. This case would occur, for example, at a longitudinal seam in a cylindrical pressure vessel. Principal stress-magnification factors for a given amount of mismatch do not vary with stress ratio if the membrane stress normal to the mismatch is greater than that parallel to the mismatch.

## INTRODUCTION

Mismatch between components of pressure vessels can result in substantial local increases in stress. Consequently, an investigation was conducted to evaluate the magnitudes of the mismatch stresses for a complete range of membrane stress ratios at a mismatched joint.

Recent utilization of high-strength materials with relatively low plane-strain fracture toughness in pressure vessels has emphasized the importance of considering stresses arising from all sources. The fabrication problems associated with increasingly larger size space boosters tend to increase the possibility of having increased local stress levels due to mismatch between the components from which the pressure vessel is man-

ufactured. Consequently, mismatch stresses and their effect on pressure vessel strength take on added importance.

The first published work on mismatch stresses known to the author (ref. 1) provided a solution for the stresses in a flat plate with a mismatched welded joint loaded in tension. The analysis was then modified in the same report to give the stresses in an axially loaded, circular cylinder of constant thickness with a mismatched circumferential joint. Reference 2 presents general linear equations in terms of edge-influence coefficients for the bending moment and shear force at a shell junction, including the effect of an axisymmetric mismatch in a thin-walled pressure vessel.

Use of the equations of reference 2 to determine the effect of mismatch on the stress distribution was verified experimentally as shown in reference 3. The cases studied were circular cylindrical pressure vessels with a continuous inner surface or a continuous outer surface at a change in thickness at a circumferential joint. Agreement between theory and experiment was excellent. More recently, a nonlinear solution for the discontinuity stresses at an axisymmetric junction between very thin shells of revolution, including the effect of mismatch at the joint, has been provided in reference 4.

Reference 5 is a brief note describing a simple, general method for determining mismatch stresses in a pressure vessel, with results given for a circumferential joint in a circular cylinder. The analysis is applicable only to shells of constant thickness, and stresses are found only at the junction. The last limitation is not deemed serious because the maximum stresses due to mismatch occur very near or at the junction. Analyses indicate that principal stresses may be a percent or so higher away from the joint under certain conditions.

In general, the stress field varies continuously along the meridian of a pressure-vessel dome. Also, flight loads and hydrostatic pressures produce variations in stress along the meridian, as well as circumferentially, in most propellant tanks. Knowing the resultant mismatch stresses in any stress field is therefore desirable.

The purpose of this report is to present equations to determine mismatch stress-magnification factors for any stress ratio and for any amount of mismatch in a pressure vessel with equal wall thicknesses on both sides of the joint. The approach described in reference 5 is extended herein to include mismatch between any two elements of equal thickness in a pressure vessel, but the joint need not be circumferential as it was in reference 5. Results are presented graphically for mismatch up to 100 percent and for stress ratios from minus infinity to plus infinity. It is recognized that the case of 100 percent mismatch appears academic, since the structure would theoretically part at the joint. In practice, however, a weld bead would probably be at such a seam to act as a transition section. Stress concentrations resulting from reduced thickness or reentrant corners are not considered in the determination of the stress-magnification factors.

In addition to a general analysis of the problem, several special cases were con-

sidered in more detail. These cases included tensile specimens, seams in pressurized spherical shells, and longitudinal and circumferential joints in cylindrical pressure vessels. The results for negative stress ratios may be useful for the analysis of pressurized shell structures carrying axial compressive loads such as the thrust loads acting on launch vehicles.

The information provided should also be of help in the selection of permissible amounts of mismatch to be included in specifications for design and manufacture of pressure vessels or other shell structures. The results are presented in a manner suitable for ready use by a designer or stress analyst.

## SYMBOLS

| | |
|---|---|
| $d$ | mismatch (distance between middle surfaces at junction), in. |
| $h$ | thickness, in. |
| $K$ | stress-magnification factor |
| $M$ | bending moment, in.-lb/in. |
| $N$ | normal force, lb/in. |
| $n$ | stress ratio, $\sigma_1/\sigma_2$ |
| $R$ | radius of curvature of joint, in. |
| $\nu$ | Poisson's ratio |
| $\sigma$ | normal stress, psi |

Subscripts:

| | |
|---|---|
| b | bending |
| cr | critical |
| d | discontinuity |
| e | effective |
| mem | membrane |
| p | principal |
| 1 | normal to mismatch |
| 2 | parallel to mismatch |

# ANALYSIS

In the ANALYSIS, equations are derived for mismatch stresses, and the figures which show a graphical presentation of the equations are mentioned. The significance of these figures is discussed in the DISCUSSION.

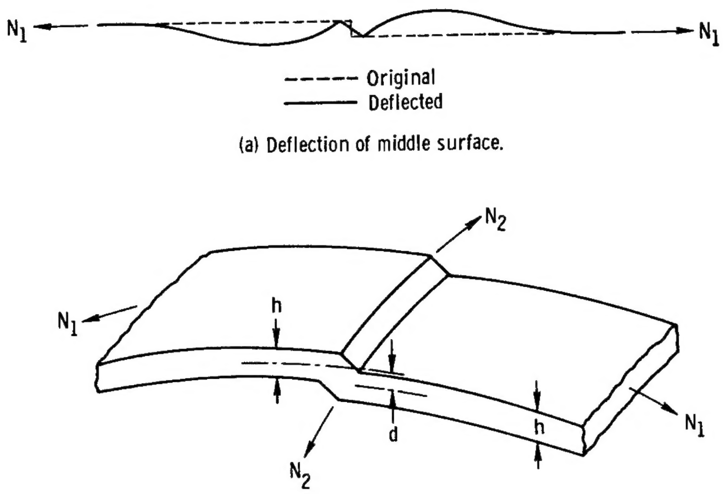
## General

The general procedure used herein for determining stresses resulting from mismatch in a pressure vessel was used first in reference 5. It is directly applicable only when the thickness and the slope on one side of a mismatched joint are the same as those on the other. Junctions between shells of different geometries with a continuous slope at the junction, as well as joints in shells of a given geometry, are thus included.

If a mismatch exists at a circumferential joint between elements of different thicknesses in a moderately thin pressure vessel, the edge-influence coefficient approach outlined in reference 2 can be used to find the stresses. For highly stressed pressure vessels with very large radius-to-thickness ratios, nonlinear equations such as presented in references 4 and 6 are necessary to find discontinuity stresses if the geometry varies across the junction. If the problem is simply mismatch between shell elements of the same thickness and slope, however, nonlinear shell equations in which the stress-magnification factor is a function of the stress level are not necessary to determine the mismatch stresses even for very thin, highly stressed pressure vessels.

The reason nonlinear shell equations are not necessary in determining mismatch stresses in constant thickness shells is that the mismatch moment is a function of mismatch and the force in the shell normal to the mismatch only. A shear force, as well as a bending moment, exists at the joint. Because the thickness is the same on both sides of the joint, a point of inflection in the deflection curve due to mismatch exists at the joint. Antisymmetry requires that the deflection due to mismatch on one side of the junction be equal and opposite to that at an equal distance on the other side as shown in figure 1(a). This antisymmetry also requires that the joint not deflect but only rotate because of the mismatch. The fact that the deflection at the joint and the force normal to the joint are the same as if no mismatch were present requires that the average stress parallel to the mismatch remain unchanged. Only uniform membrane stress and discontinuity bending stress due to mismatch are thus present at the joint. The mean stresses parallel and perpendicular to the mismatch at the joint are not affected by the mismatch moment or shear force.

In this report, only simple beam formulas, without the use of edge-influence coefficients, are required for the solution. The forces and stresses only at the mismatch




(a) Deflection of middle surface.



(b) Loading and geometry.

Figure 1. - Mismatched joint in shell.

are discussed. The method of analysis used does not furnish any information about the distribution of forces away from the joint. Since the stresses contributed by the mismatch are usually maximum at or very near the joint, the information furnished herein is believed to be all that is necessary for most stress analysis purposes.

Analyses of axially symmetric mismatched joints performed with the use of edge-influence coefficients indicate that principal stresses associated with mismatch may be a percent or so higher at a slight distance from the joint. These stresses may not be a maximum right at the joint because the bending stresses due to the mismatch shear force may increase somewhat faster than those due to the mismatch bending moment decrease. However, the error involved in neglecting this small increase in stress which may occur is believed to be negligible for engineering purposes.

If a change in curvature also occurs at the mismatch, additional discontinuity stresses result from this change. The mismatch stresses can be superimposed upon the geometrical discontinuity stresses and the membrane stresses to obtain the total stress.

## Basic Equations

As shown in figure 1(b), $N_1$ is the membrane force per unit length normal to the mismatch in the shell and $N_2$ is the force per unit length parallel to the mismatch.

The corresponding membrane stresses are

$$\sigma_{1,\mathrm{mem}} = \frac{N_1}{h} \tag{1a}$$

and

$$\sigma_{2,\,mem} = \frac{N_2}{h} \tag{1b}$$

respectively. The bending moment at the joint due to the eccentricity of the middle surfaces or mismatch is

$$M_d = N_1 d \tag{2}$$

The effect of this bending moment is the same as if an external moment of the same magnitude were uniformly distributed along the junction which has this mismatch. One-half of the mismatch moment is applied to the shell on each side of the joint because the thicknesses, and therefore stiffnesses, are assumed to be identical. The bending stress in the direction normal to the mismatch is, thus,

$$\sigma_{1,\,b} = \pm \frac{6M_1}{h^2} = \pm \frac{6}{h^2}\frac{M_d}{2} = \pm \frac{3N_1 d}{h^2} = \pm\, 3\frac{d}{h}\sigma_{1,\,mem} \tag{3a}$$

If the mismatch is assumed to be relatively long, little change in the curvature of the shell can take place in a direction parallel to the mismatch. Consequently, the bending stress parallel to the mismatch is

$$\sigma_{2,\,b} = \nu\sigma_{1,\,b} = \pm 3\nu\frac{d}{h}\sigma_{1,\,mem} \tag{3b}$$

where $\nu$ is Poisson's ratio. The resultant stresses are then

$$\sigma_1 = \sigma_{1,\,mem} + \sigma_{1,\,b} = \left(1 \pm 3\frac{d}{h}\right)\sigma_{1,\,mem} \tag{4a}$$

and

$$\sigma_2 = \sigma_{2,\,mem} + \sigma_{2,\,b} = \sigma_{2,\,mem} \pm 3\nu\frac{d}{h}\sigma_{1,\,mem} \tag{4b}$$

Let the membrane stress normal to the mismatch be n times the membrane stress parallel to the mismatch

$$\sigma_{1, mem} = n\sigma_{2, mem} \tag{5}$$

Then equations (4a) and (4b) become

$$\sigma_1 = \left(1 \pm 3\frac{d}{h}\right)n\sigma_{2, mem} \tag{6a}$$

and

$$\sigma_2 = \left(1 \pm 3\nu n\frac{d}{h}\right)\sigma_{2, mem} \tag{6b}$$

## Principal Stresses

In determining critical stresses and stress-magnification factors, the stress of largest absolute magnitude is considered critical, whether it be tensile or compressive. Thus, only the positive sign need be used in equation (6a) when critical principal stresses are found for positive or negative n. Thus,

$$\sigma_{1, cr} = \left(1 + 3\frac{d}{h}\right)n\sigma_{2, mem} \tag{6a1}$$

In equation (6b), the positive sign should be used with positive n and the negative sign with negative n in determining critical principal stresses. For positive n, the critical principal stress is

$$\sigma_{2, cr} = \left(1 + 3\nu n\frac{d}{h}\right)\sigma_{2, mem} \tag{6b1}$$

For negative n,

$$\sigma_{2, cr} = \left(1 - 3\nu n\frac{d}{h}\right)\sigma_{2, mem} \tag{6b2}$$

Depending upon the value of n, $\sigma_{1, mem}$ may be greater than, equal to, or less than $\sigma_{2, mem}$. By use of equations (5), (6a1), (6b1), and (6b2) the principal stress-magnification factors perpendicular and parallel to the mismatch are, respectively,

Positive or negative n:

$$\frac{\sigma_1}{\sigma_{1,\,mem}} = 1 + 3\frac{d}{h} \tag{7}$$

Positive n:

$$\frac{\sigma_2}{\sigma_{2,\,mem}} = 1 + 3\nu n\frac{d}{h} \tag{8a}$$

Negative n:

$$\frac{\sigma_2}{\sigma_{2,\,mem}} = 1 - 3\nu n\frac{d}{h} \tag{8b}$$

Positive stress ratio n. - For design purposes, only the larger of the two principal stresses is usually of interest. If n is between 1 and -1, the numerically maximum membrane stress is parallel to the mismatch. Then, from equations (5), (7), and (8a), for positive stress ratios n the principal stress-magnification factors $K_p$ are

$0 < n < 1$:

$$K_{p,\,1} = \frac{\sigma_1}{\sigma_{2,\,mem}} = \frac{\sigma_1}{\sigma_{1,\,mem}}\,n = n\left(1 + 3\frac{d}{h}\right) \tag{9}$$

$$K_{p,\,2} = \frac{\sigma_2}{\sigma_{2,\,mem}} = 1 + 3\nu n\frac{d}{h} \tag{8a}$$

A comparison of equations (8a) and (9) shows that for large mismatch and biaxiality (d/h and n approaching 1) the larger stress is perpendicular to the mismatch. For smaller mismatch and biaxiality, however, the maximum stress is parallel to the mismatch. Solving equations (8a) and (9) for n gives

$$n = \frac{1}{1 + 3(1 - \nu)\frac{d}{h}} \tag{10}$$

Figure 2 (p. 16) is a plot of this equation and shows the region in which equation (8a) or (9) is critical.

For n greater than 1, the maximum membrane stress is perpendicular to the mismatch. Then, from equations (5), (7), and (8a),

$n > 1$:

$$K_{p,1} = \frac{\sigma_1}{\sigma_{1,\,mem}} = 1 + 3\frac{d}{h} \tag{7}$$

$$K_{p,2} = \frac{\sigma_2}{\sigma_{1,\,men}} = \frac{\sigma_2}{n\sigma_{2,\,mem}} = \frac{1 + 3\,\nu n \frac{d}{h}}{n} \tag{11}$$

If equations (7) and (11) are compared, it can be seen that the maximum stress is always perpendicular to the mismatch for n greater than 1 as is to be expected. A plot of maximum principal stress-magnification factor as a function of mismatch and stress ratio is presented in figure 3(a) (p. 17) for relatively long mismatches.

If the mismatch is relatively short in length (less than about 5 characteristic lengths $\sqrt{Rh}$), local distortions will occur which relieve mismatch stresses parallel to the mismatch. These local distortions arise because complete radial constraint requiring radial planes to remain so is not present for local deviations from proper contour. The tendency is to induce strains because of Poisson's effect but not stresses. Thus, Poisson's ratio should be set equal to zero in equation (8a) for the case of local mismatch. The maximum principal stress-magnification factor for local mismatch is plotted as figure 3(b) (p. 17). Note that the maximum stress in the structure can never be less than the membrane stress. Thus, the minimum stress-magnification factor is 1.

Principal stress-magnification factors for a circumferential joint in a cylindrical pressure vessel are given in figure 4 (p. 18). Those for a mismatched joint in a uniaxially stressed member, a joint in a spherical pressure vessel, or a longitudinal seam in a cylindrical pressure vessel are the same and are shown in figure 5 (p. 18). These figures are simply cross plots from figure 3 (p. 17).

Negative stress ratio n. - If n is negative, equation (8b) is used in place of equation (8a) in determining stress-magnification factors. As mentioned previously, if n is between 1 and -1, the numerically maximum membrane stress is parallel to the mismatch. Using equations (5), (7), and (8b) yields

$-1 < n < 0$:

$$K_{p,1} = \frac{\sigma_1}{\sigma_{2,\,mem}} = n\left(1 + 3\frac{d}{h}\right) \tag{9}$$

$$K_{p,2} = \frac{\sigma_2}{\sigma_{2,\,mem}} = 1 - 3\nu n\frac{d}{h} \tag{8b}$$

In determining which principal stress is critical, only the absolute magnitudes of equations (8b) and (9) are of interest. Since n is negative for this case, an equation identical to equation (10) results when the absolute values of equations (8b) and (9) are solved for n. Therefore, figure 2 (p. 16) also separates the regions in which equation (8b) or (9) is critical.

For n between -1 and $-\infty$, the membrane stress with the maximum absolute value is perpendicular to the mismatch. Then, from equations (5), (7), and (8b)

$-\infty < n < -1$:

$$K_{p,1} = \frac{\sigma_1}{\sigma_{1,\,mem}} = 1 + 3\frac{d}{h} \tag{7}$$

$$K_{p,2} = \frac{\sigma_2}{\sigma_{1,\,mem}} = \frac{1 - 3\nu n\frac{d}{h}}{n} \tag{12}$$

As for the case where n is greater than 1, the stress with the larger absolute value is always perpendicular to the mismatch for n less than -1.

As a result of the foregoing analysis, figure 3 (p. 17) can be used to find the principal stress magnification factors for both positive and negative n. These factors are based on the assumption that the stress larger in absolute magnitude, whether it be tensile or compressive, is critical.

## Effective Stresses

Maximum effective stress. - If the distortion energy theory is used as a yield criterion, the effective stress is

$$\sigma_e = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \tag{13}$$

Substitution of equations (6a) and (6b) into equation (13) gives effective stresses including the effect of mismatch. In finding maximum effective stresses, the surface with compressive bending stresses must be considered in addition to the surface with tensile bending stresses. When equations (6a) and (6b) are substituted into equation (13), positive bending parallel to the mismatch is associated with positive bending perpendicular to the mismatch. Similarly, the negative signs are used simultaneously when compressive stresses are considered.

Performing the previously mentioned substitution results in the following equation for effective stress at the mismatch on the surface with tensile bending stresses (indicated by (+)):

$$\sigma_e(+) = \sigma_{2,\,mem}\left\{(n^2 - n + 1) + 3n\Big[(2n - 1) + \nu(2 - n)\Big]\left(\frac{d}{h}\right) + 9n^2(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2\right\}^{1/2} \tag{14a}$$

Similarly, the effective stress on the surface with compressive bending stresses (indicated by (-)) is

$$\sigma_e(-) = \sigma_{2,\,mem}\left\{(n^2 - n + 1) - 3n\Big[(2n - 1) + \nu(2 - n)\Big]\left(\frac{d}{h}\right) + 9n^2(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2\right\}^{1/2} \tag{14b}$$

Equations (14a) and (14b) must be compared to determine the critical effective stress. Solving these two equations simultaneously results in $n = 0$ and

$$n = \frac{1 - 2\nu}{2 - \nu} \tag{15}$$

For mismatched joints with stress ratios between zero and the value given by equation (15), the maximum effective stress is on the surface with compressive bending stress and is given by equation (14b). Conversely, for all other stress ratios, the maximum effective stress is on the surface with tensile bending stresses and is given by equation (14a).

The effective stress in the membrane region is

$$\sigma_{e,\,mem} = \sigma_{2,\,mem}\sqrt{n^2 - n + 1} \tag{16}$$

The effective stress magnification factors $K_e$ associated with the mismatch are therefore obtained by dividing equations (14a) and (14b) by equation (16). The resulting equations are

$$K_e(+) = \frac{\sigma_e}{\sigma_{e,\,mem}}(+) = \left\{1 + \frac{3n}{n^2 - n + 1}\frac{d}{h}\left[3n(1 - \nu + \nu^2)\frac{d}{h} + 2n - 1 + \nu(2 - n)\right]\right\}^{1/2} \tag{17a}$$

and

$$K_e(-) = \frac{\sigma_e}{\sigma_{e,\,mem}}(-) = \left\{1 + \frac{3n}{n^2 - n + 1}\frac{d}{h}\left[3n(1 - \nu + \nu^2)\frac{d}{h} - 2n + 1 - \nu(2 - n)\right]\right\}^{1/2} \tag{17b}$$

<u>Stress-magnification factors for long mismatch.</u> - For $\nu = 0.3$, equation (15) gives $n = 0.235$ as the stress ratio above which the maximum effective stress is on the surface with tensile bending. Equation (17a) therefore gives the stress-magnification factors for stress ratios greater than 0.235 and less than zero if Poisson's ratio is 0.3. In a like manner, equation (17b) gives stress-magnification factors for stress ratios less than 0.235 but greater than zero.

Effective stress-magnification factors for long or continuous mismatch ($\nu = 0.3$) are given in figures 6(a) and (b) (p. 19) for positive and negative stress ratios, respectively.

<u>Stress-magnification factors for local mismatch.</u> - If a mismatched joint is axially symmetric, symmetry and continuity require that radial planes remain radial. As is well known from familiar discontinuity analyses, circumferential bending moments and stresses equal to Poisson's ratio times the corresponding meridional values are induced. This same tendency exists for relatively long mismatched joints oriented in any direction, as was assumed in equation (3b). However, as mentioned previously, if the mismatch is relatively short in length, local distortions will occur. Then the tendency is to induce strains because of Poisson's effect but not stresses.

An approximate effective stress-magnification factor for local mismatch can therefore be obtained simply by setting Poisson's ratio equal to zero in equations (17); the following equations result:

$$K_e(+) = \frac{\sigma_e}{\sigma_{e,\,mem}}(+) = \left[1 + \frac{3n}{n^2 - n + 1}\frac{d}{h}\left(3n\frac{d}{h} + 2n - 1\right)\right]^{1/2} \tag{18a}$$

and

$$K_e(-) = \frac{\sigma_e}{\sigma_{e,\,mem}}(-) = \left[1 + \frac{3n}{n^2 - n + 1}\frac{d}{h}\left(3n\frac{d}{h} - 2n + 1\right)\right]^{1/2} \tag{18b}$$

Substitution of $\nu = 0$ into equation (15) gives $n = 1/2$. The maximum effective stress-magnification factor therefore occurs on the surface with compressive bending stresses and is given by equation (18b) for stress ratios between zero and 1/2 for local mismatch. It occurs on the tensile bending stress surface for all other values of stress ratio n. Effective stress-magnification factors for local mismatch obtained from equations (18) are presented in figures 6(c) and (d) (p. 20) for positive and negative stress ratios, respectively.

Special cases. - Equation (17a) was solved for several frequently occurring cases, and the results are as follows:

n = 0 (Uniaxial load parallel to mismatch):

$$(K_e)_{n=0} = 1 \tag{19}$$

n = 1/2 (Circumferential joint in cylindrical pressure vessel):

$$(K_e)_{n=1/2} = \left[1 + 3\nu\frac{d}{h} + 3(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2\right]^{1/2} \tag{20}$$

n = 1 (Mismatched joint in spherical pressure vessel):

$$(K_e)_{n=1} = \left[1 + 3(1 + \nu)\frac{d}{h} + 9(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2\right]^{1/2} \tag{21}$$

n = 2 (Longitudinal joint in cylindrical pressure vessel):

$$(K_e)_{n=2} = \left[1 + 6\frac{d}{h} + 12(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2\right]^{1/2} \tag{22}$$

$n = \infty$ (Uniaxial load perpendicular to mismatch):

$$(K_e)_{n=\infty} = \left[1 + 3(2 - \nu)\frac{d}{h} + 9(1 - \nu + \nu^2)\left(\frac{d}{h}\right)^2\right]^{1/2} \tag{23}$$

The last equation was obtained by successive applications of L'Hospital's rule, made necessary by the indeterminate form of the equation when $n$ is infinite. A simple check can be made of equation (23) if the tensile specimen is narrow. Setting $\nu$ equal to zero gives

$$(K_e)_{n=\infty, \nu=0} = 1 + 3\frac{d}{h}$$

which compares with equation (4a). Equations (20) to (23) are plotted as figures 7(a) to (d), respectively (p. 21).

# DISCUSSION

## General

Stress-magnification factors have been determined in this report for shells of constant thickness with varying amounts of mismatch and for any biaxial stress field. Because of the method of analysis used, it was not necessary to use edge-influence coefficients. In fact, the usual edge-influence coefficient approach is normally used only for axially symmetric joints, whereas the method used herein is applicable for almost any joint in a shell.

The results are a function of dimensionless mismatch, stress ratio, and Poisson's ratio only. Contrary to the usual solutions for shell problems, the curvature of the shell does not enter into the equations or affect the results. Also, as discussed in the ANALYSIS section, because shells of constant thickness are being considered, nonlinear shell solutions are unnecessary even though the shells may have high radius-to-thickness ratios and be highly stressed.

All curves of stress-magnification factors given herein are presented for two different conditions, namely, long or local mismatch. A mismatch is considered local if its length is less than about 5 characteristic lengths $\sqrt{Rh}$. For local mismatch, local distortions due to Poisson strains are assumed to take place to relieve stresses parallel to the mismatch. For long or continuous mismatch, however, such as at an axially symmetric joint, changes in curvature parallel to the mismatch cannot take place. The simple procedure used to find the stresses for the case of local mismatch was to set

Poisson's ratio equal to zero in the equations which had been derived for cases of long or continuous mismatch.

## Stress Distribution

As with the more usual solutions for discontinuity stresses in shells, the stresses resulting from mismatch should decay exponentially with distance normal to the joint. The effect of the mismatch is thus felt for several characteristic lengths on either side of the joint. If it is desired to know the distribution of stress away from the joint to a fairly high degree of accuracy, it may be necessary to use finite deflection theory, particularly if the shell is very thin and highly stressed.

Mismatch stresses are antisymmetric on either side of a joint; that is, stresses on the outer surface on one side of the joint are the same as those on the inner surface at an equal distance on the other side of the joint if the thickness is constant.

Only stresses at the joint are given herein. These stresses are essentially the maximum stresses and, thus, usually all that is of interest to the designer. Occasions do arise, however, when it is desirable to know the distribution of stress away from the mismatch. Some work with regard to this subject is presently being done at the Boeing Company. In addition, they are also investigating angular mismatch, that is, an imperfection which is a discontinuity in slope such as a cusp or a "sink-in" at a joint.

## Principal Stresses

Some materials are subject to brittle failure even though the basic stress state may be elastic. For such situations, principal stress-magnification factors are usually more useful than effective stress-magnification factors. Principal stress-magnification factors may be particularly useful when mismatch stresses in pressure vessels are being considered, because most joints or seams in pressure vessels are either circumferential or along a meridian and thus parallel to the principal stresses.

Depending upon the stress ratio and the amount of mismatch, the maximum principal stress may be either parallel or perpendicular to the mismatch. A plot of the parameters bounding the regions of different directions of maximum principal stress is given in figure 2. For relatively low values of the stress ratio $n$ and mismatch $d/h$, the maximum principal stress is parallel to the mismatch. This situation arises because of the overriding influence of the maximum membrane stress which is parallel to the mismatch for $n$ less than 1. Conversely, values above and to the right of the appropriate curve indicate maximum stresses perpendicular to the mismatch.

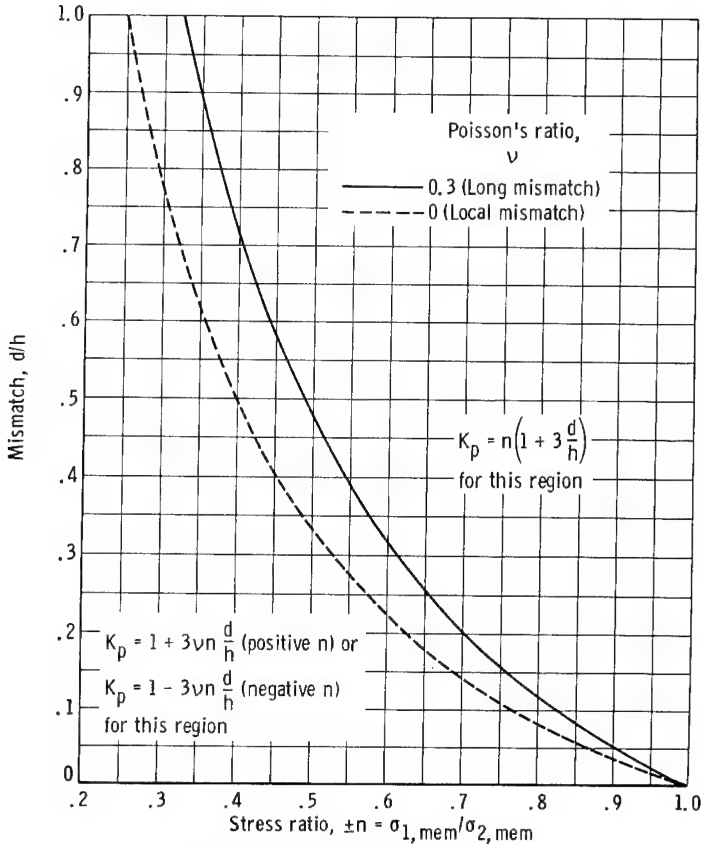
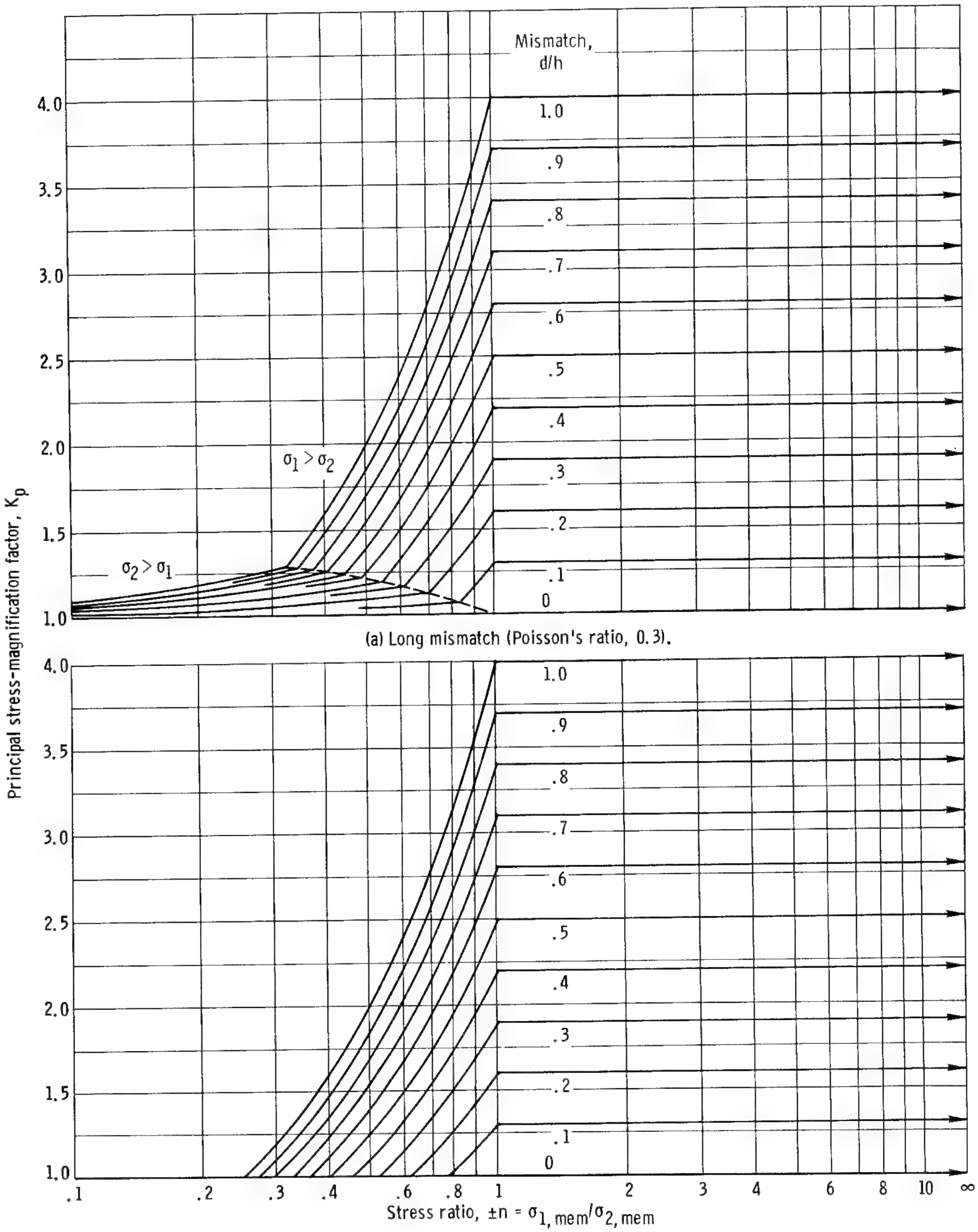



Figure 2. - Parameters defining direction of maximum principal stress.

Plots of principal stress-magnification factor for long and local mismatch are presented in figures 3(a) and (b), respectively, for both positive and negative stress ratios n. Note the similarity of the two figures. The only difference is that the stress-magnification factor is 1 for local mismatch for small values of n, whereas it is greater than 1 for long mismatch in all instances. Actually, for intermediate lengths of mismatch, this factor lies somewhere between 1 and the value given in figure 3(a) in the region where the stress parallel to the mismatch $\sigma_2$ is critical. The line of demarcation between the region where the maximum principal stress is perpendicular to the mismatch and that where it is parallel to the mismatch should be noted. Stress-magnification factors are the same for positve and negative n for the same percentage mismatch.

Values of n to the left of the intersection of the appropriate mismatch curve with the abscissa axis in figure 3(b) have maximum stresses parallel to the mismatch and thus a principal stress-magnification factor of 1. Shells with stress ratios and mismatch in this region thus have no increase in the maximum stress due to the local mismatch, since it is assumed that there exists no stress due to the Poisson effect. Note that principal stress-magnification factors for a given percentage mismatch are the same for all shells in which the membrane stresses normal to the mismatch are equal to or greater




(a) Long mismatch (Poisson's ratio, 0.3).

(b) Local mismatch (Poisson's ratio, 0).

Figure 3. - Principal stress-magnification factors for mismatch.

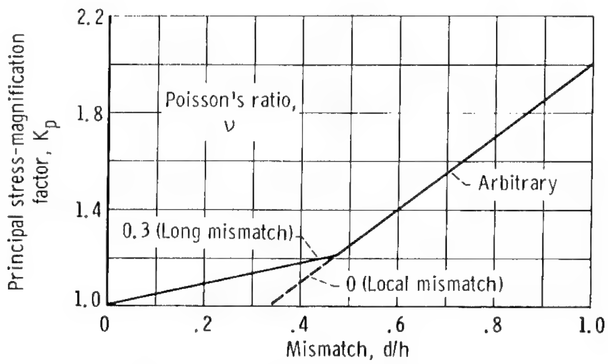



Figure 4. - Principal stress-magnification factors for circumferential joint in cylindrical pressure vessel. Stress ratio, 1/2.

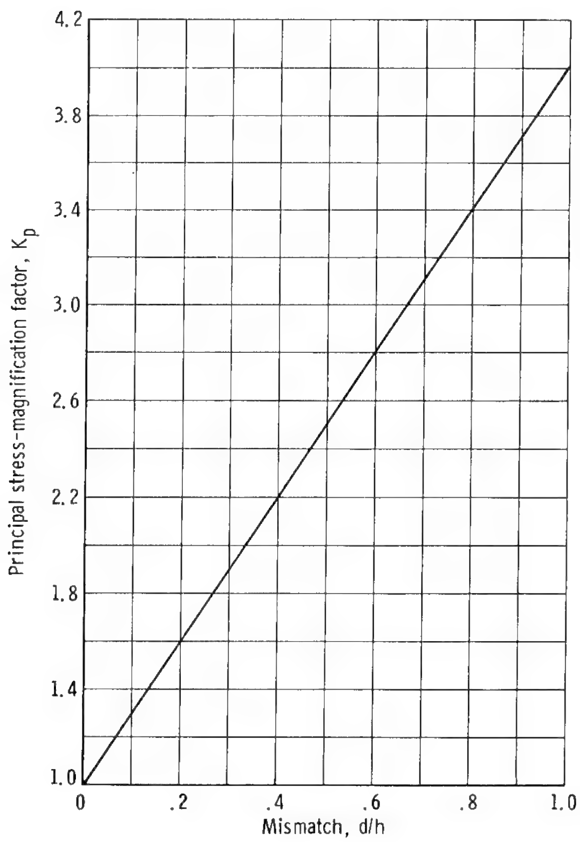



Figure 5. - Principal stress-magnification factors for stress ratios greater than 1 (includes uniaxial stress field, joint in spherical pressure vessel, and longitudinal joint in cylindrical pressure vessel). Poisson's ratio arbitrary.

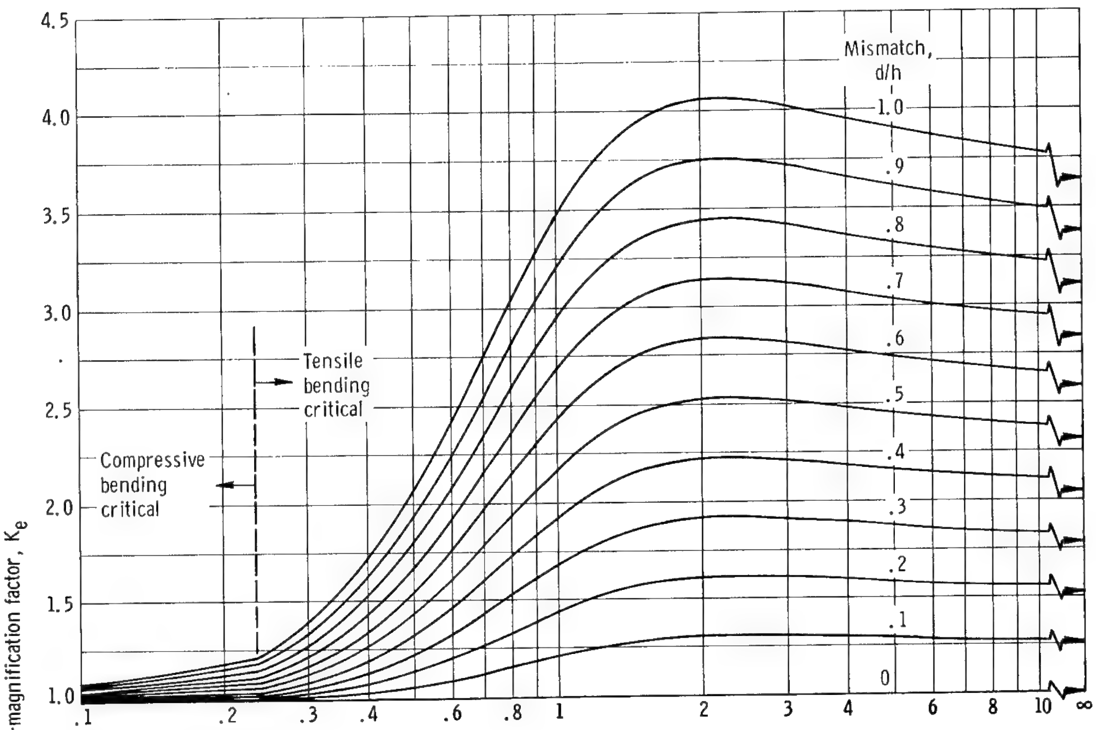
than the membrane stresses parallel to the mismatch.

Figures 4 and 5 are cross plots for some of the data in figure 3. Figure 4 presents principal stress-magnification factors for a circumferential joint in a cylindrical pressure vessel. Principal stress-magnification factors for all cases in which the membrane stress normal to the mismatch is equal to or greater than that parallel to it are presented in figure 5. These factors are good for both long and local mismatch, because the stresses induced by the Poisson effect do not enter into the calculations.

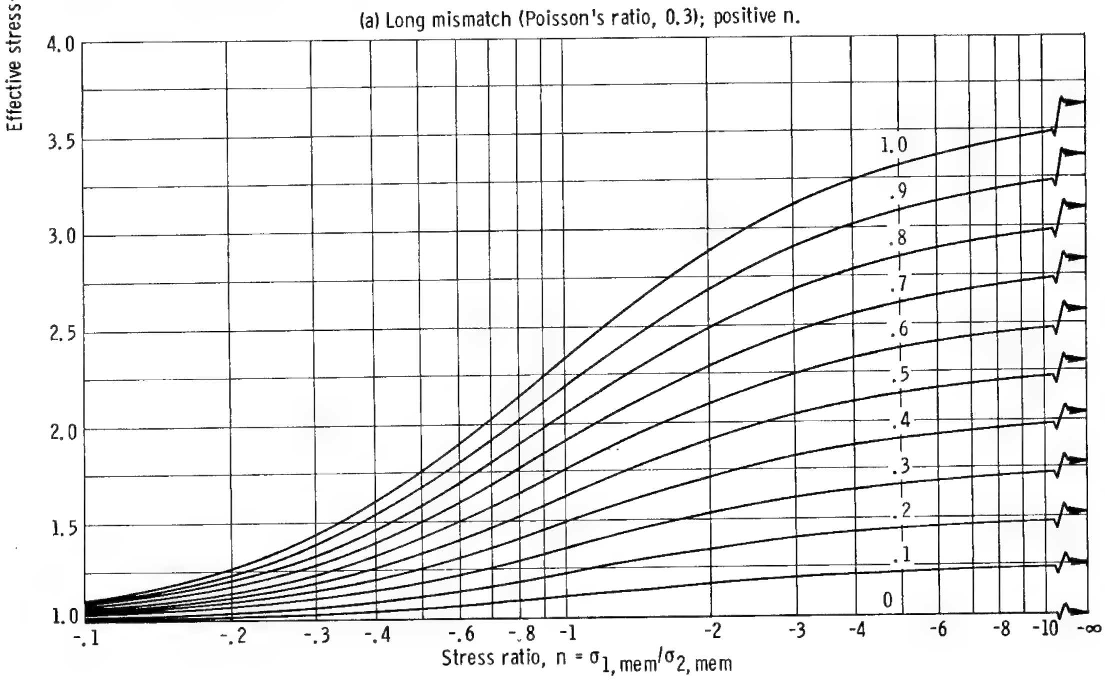
## Effective Stresses

The maximum distortion energy theory is perhaps the most widely accepted yield criterion. For this reason, stress-magnification factors are given herein in terms of effective stresses based on the distortion-energy theory as well as principal stresses. For a material with a reasonable amount of ductility, effective stress-magnification factors can thus be used to predict onset of plastic flow.

Effective stress-magnification factors are presented in figure 6 as a function of stress ratio for various amounts of mismatch. Figures 6(a) and (b) are for long mismatch, and figures 6(c) and (d) are for local mismatch. Note that the stress-magnification factor increases rapidly with increasing stress ratio for positive n to a peak at a stress ratio of about 2. A stress ratio of 2 corresponds to a longitu-

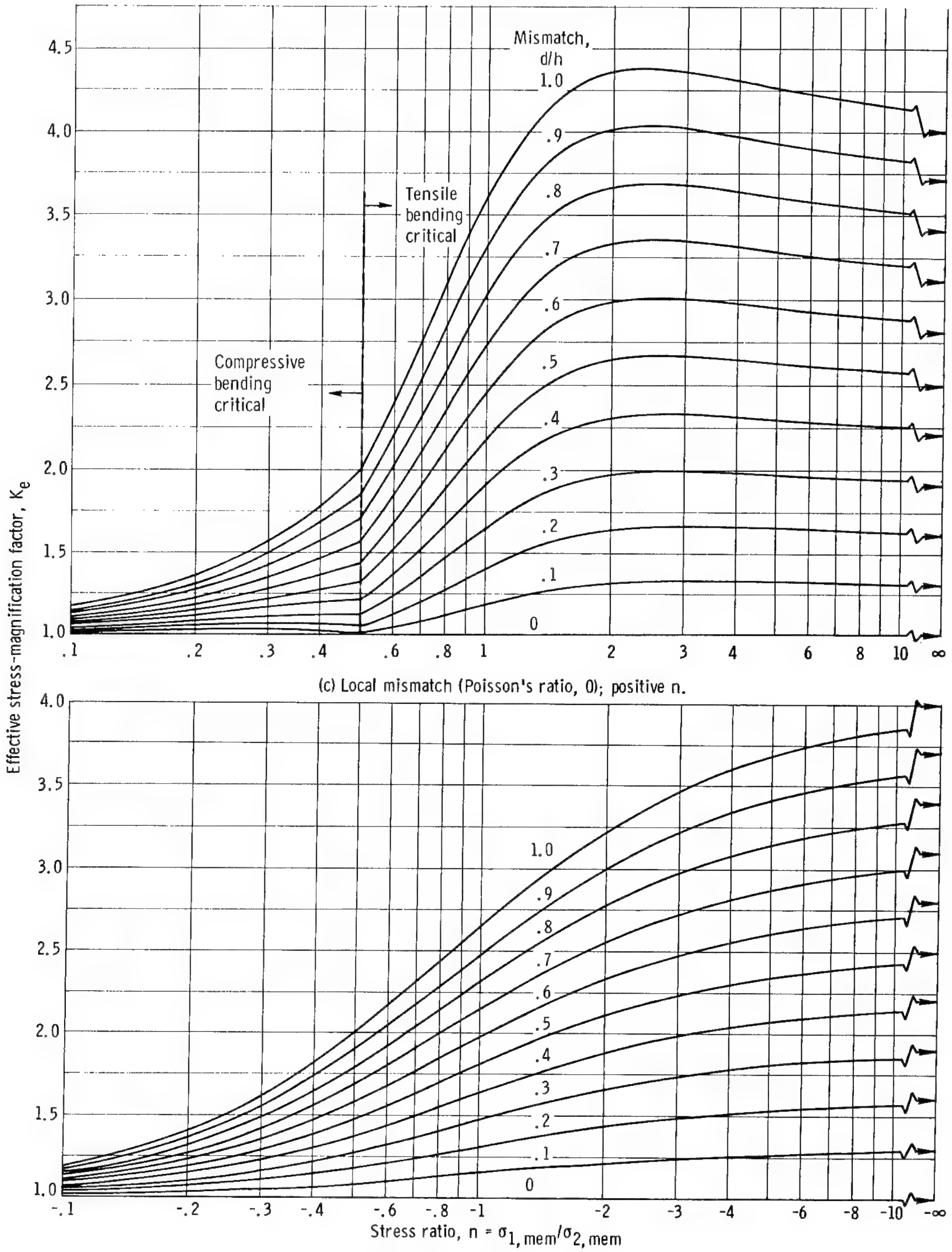



(a) Long mismatch (Poisson's ratio, 0.3); positive n.




(b) Long mismatch (Poisson's ratio, 0.3); negative n.

Figure 6. - Effective stress-magnification factors for mismatch.




(c) Local mismatch (Poisson's ratio, 0); positive n.

(d) Local mismatch (Poisson's ratio, 0); negative n.

Figure 6. - Concluded.

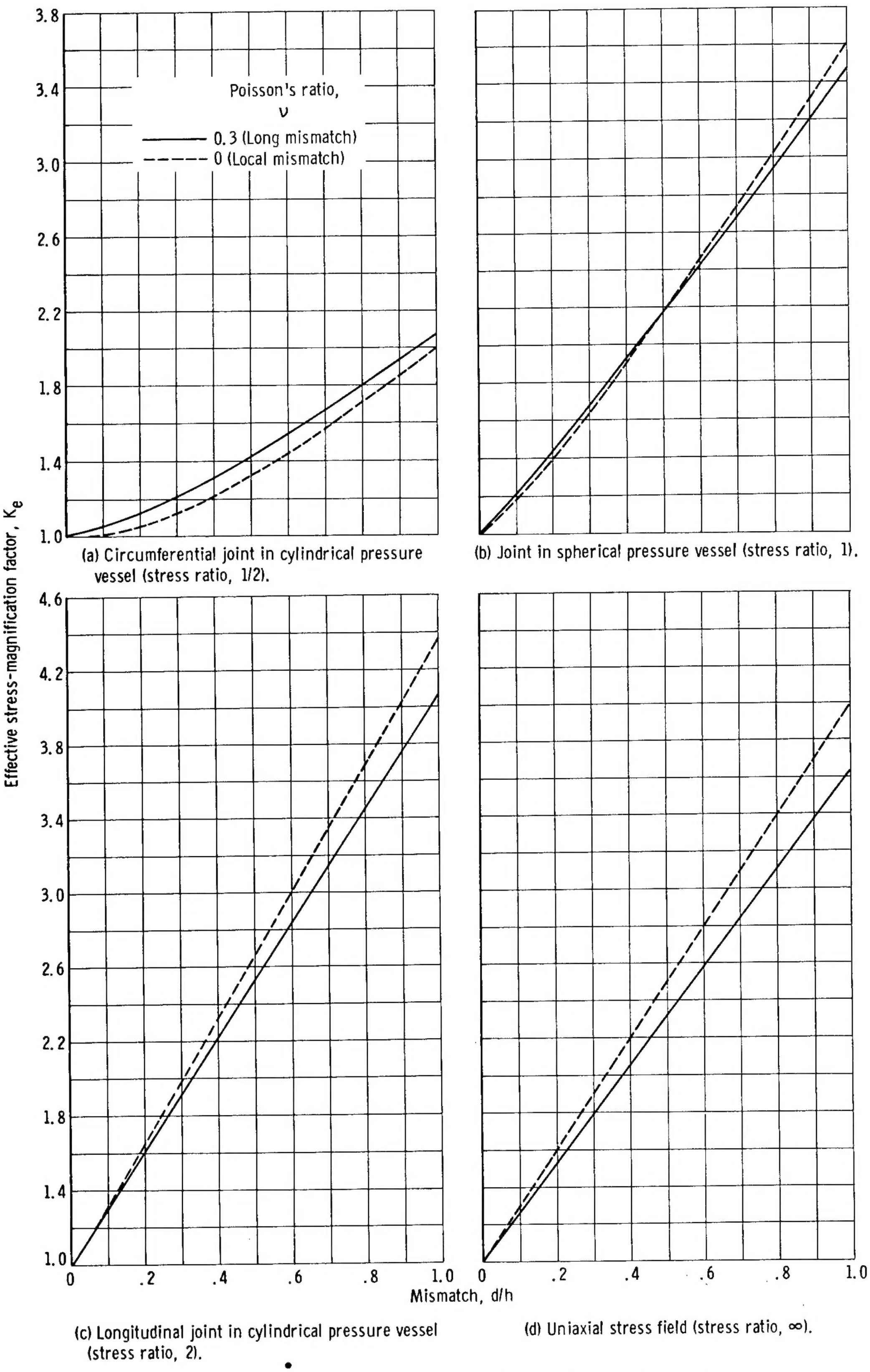



Figure 7. - Effective stress-magnification factors for several frequently occurring cases.

dinal seam in a cylindrical pressure vessel. Mismatch stresses are very sizable in magnitude, particularly for this case. For $n = 2$ and 10 percent mismatch, for example, bending effective stresses equal to 30 percent of the membrane effective stress occur. By contrast, 10 percent mismatch in the circumferential seam of a cylindrical pressure vessel produces only 5 percent increase in stress. In further comparison, 10 percent mismatch in a sphere produces about 20 percent bending, and 10 percent mismatch in a uniaxial specimen gives rise to bending stresses of about 26 percent of the nominal stress. These numbers are for long mismatch and are somewhat different for local mismatch. Note that effective stress-magnification factors for local mismatch are higher than for long mismatch for all negative stress ratios. Also, for a given stress ratio, the increase in stress is almost proportional to the amount of mismatch, with the effective stress increasing at a faster rate for greater amounts of mismatch.

Figure 7 presents effective stress-magnification factors for several common types of joints in pressure vessels. Effective stress-magnification factors for a circumferential joint in a cylindrical pressure vessel are shown in figure 7(a). Bending stresses associated with local mismatch are smaller than for long mismatch for small values of d/h. The stress magnification factors for both long and local mismatch, however, are about the same over the entire range of d/h. Note that these factors are larger than the principal stress-magnification factors given in figure 4 for the same condition.

The influence of mismatch on the stress state in a spherical pressure vessel is shown in figure 7(b). There is very little difference between long and local mismatch; for less than 50 percent mismatch, a long mismatch is only slightly more severe than one that is local.

Figure 7(c) is of considerable interest because it is concerned with longitudinal seams in cylindrical pressure vessels. Bending stresses are almost linear with amount of mismatch for this case; also, a local mismatch is somewhat worse than a long one.

The simple case of mismatch in a uniaxially stressed member is presented in figure 7(d). Although the curves in figures 7(c) and (d) appear to be linear, examination of equations (22) and (23), which were used to plot these figures, shows that they are not. There is a significant difference between local and long mismatch, with the former being worse than the latter in a uniaxially loaded member.

## CONCLUSIONS

From a study of theoretical elastic mismatch stresses, was made and the following conclusions were drawn. conclusions They apply only to mismatch between shell elements with the same thickness and slope at the joint.

1. The principal stress-magnification factors for a given percentage mismatch are

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the same for all shells in which the membrane stresses normal to the mismatch are equal to or greater than the membrane stresses parallel to the mismatch.

2. Principal stress-magnification factors are the same for positive and negative stress ratios for a given percentage mismatch.

3. In the consideration of effective stresses, mismatch is most serious if the membrane stress normal to the mismatch is about twice that parallel to the mismatch, such as at a longitudinal seam in a circular cylindrical pressure vessel. The stress-magnification factor is quite sizable, however, whenever the membrane stress normal to the mismatch is about equal to or greater than that parallel to the mismatch.

4. In areas in which the membrane stress normal to the mismatch is much less than the stress parallel to the mismatch, a small amount of mismatch may not increase the stress significantly, if at all.

5. For a given stress ratio, the increase in stress is almost proportional to the amount of mismatch, with the effective stress increasing at a faster rate for greater amounts of mismatch.

6. A local mismatch has a higher effective stress-magnification factor than a long mismatch for all negative stress ratios. # end

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, October 15, 1965.

## REFERENCES


1. Sechler, E.E.: Stress Rise Due to Offset Welds in Tension. Rept. No. EM 9-18 (TR-59-0000-00774), Space Technology Labs., Inc., Aug. 28, 1959.

2. Johns, Robert H.; and Orange, Thomas W.: Theoretical Elastic Stress Distributions Arising from Discontinuities and Edge Loads in Several Shell-Type Structures. NASA TR R-103, 1961, Appendix L.

3. Johns, R.H.; Morgan, W.C.; and Spera, D.A.: Analysis of Stress at Several Junctions in Pressurized Shells. AIAA J., vol. 1, no. 2, Feb. 1963, pp. 455-457.

4. Wittrick, W.H.: Non-Linear Discontinuity Stresses in Shells of Revolution Under Internal Pressure. Int. J. Eng. Sci., vol. 2, May 1964, pp. 179-188.

5. Johns, Robert H.: Mismatch Stresses in Pressure Vessels. AIAA J., vol. 2, no. 10, Oct. 1964, pp. 1827-1828.

6. Smith, George W.: Analysis of Multiple Discontinuities in Shells of Revolution Including Coupled Effects of Meridional Load. Rept. No. GD/A 63-0044, General Dynamics/Astronautics, July 31, 1963.

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